Spatial Variation of Sea-Level

Sea level reconstruction

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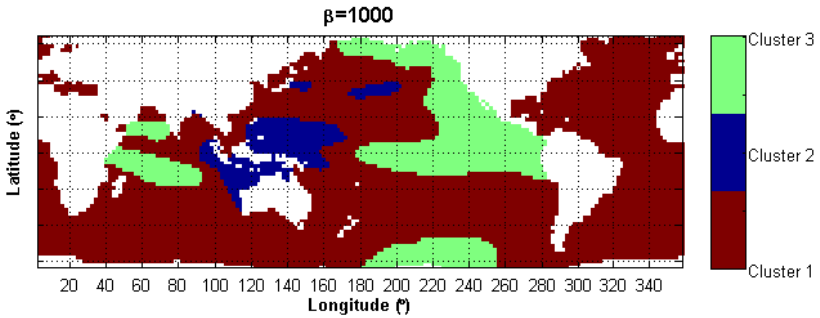
Problem definition

❑ Recap of last presentation

- Fuzzy C-Means
- Non-spherical shapes in the attribute domain
- Spatial contiguity in the geographic domain
- Mean silhoutte value → number of clusters



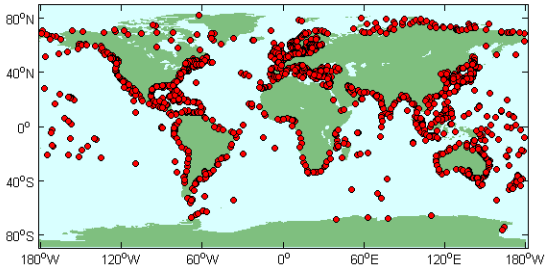
❑ Subsequent work

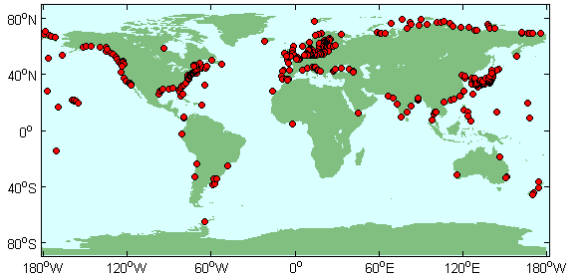
- Semi-empirical modeling
- Definition of "empirical":

  adj. based on, concerned with, or verifiable by observation or experience rather than theory or pure logic (http://oxforddictionaries.com/us/definition/american_english/empirical)

❑ Observation shortage

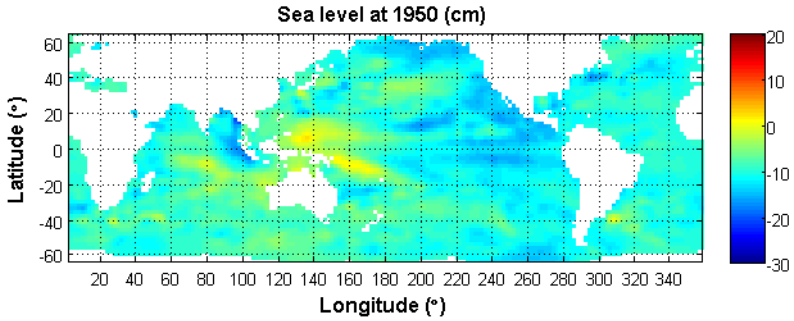
- Temporal coverage of spatial sea level data: 1950 to 2001
- Tide gauge stations (observational data):

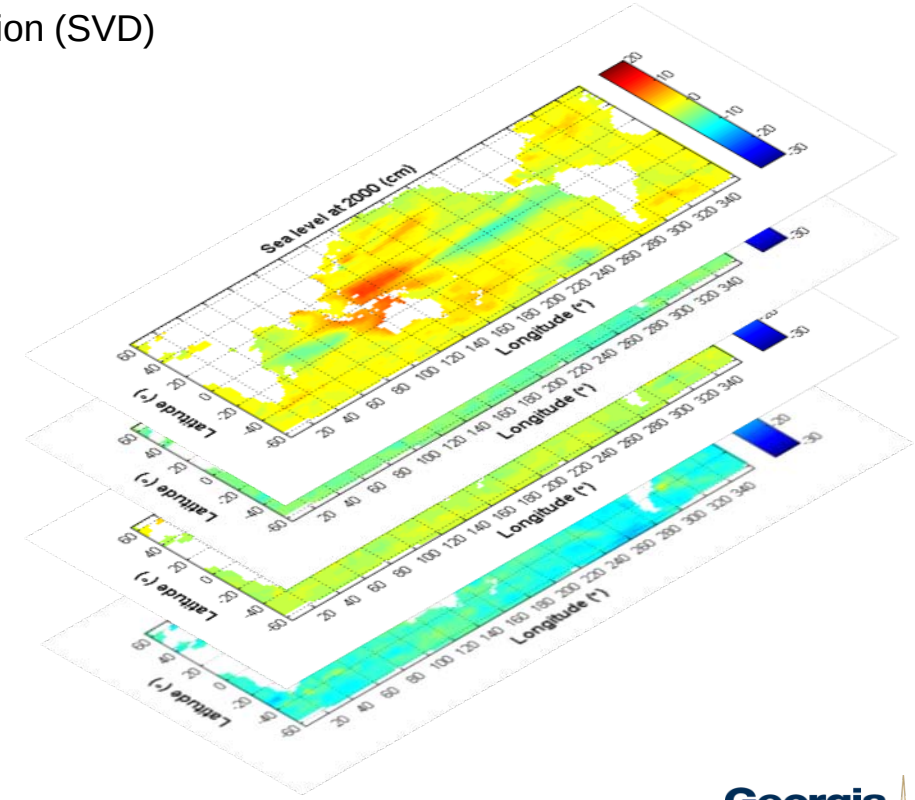




Methodology of reconstruction

- The basic ideas
  - Filter out noise
  - Capture spatial pattern
  - Fill data gaps
- Review of previous methods
  - Originated from Singular Value Decomposition (SVD)





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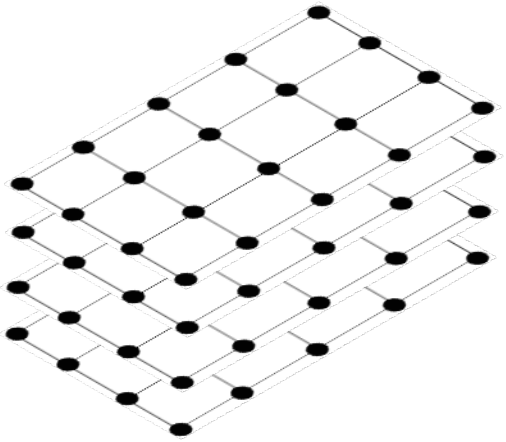

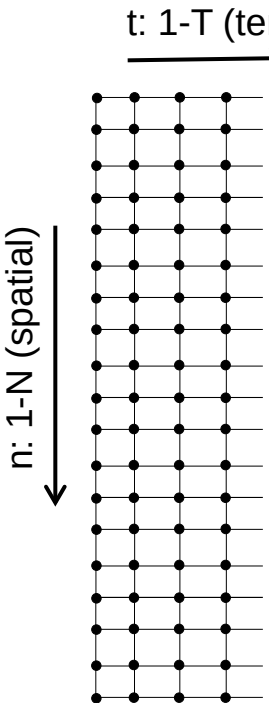

Sea level data matrix **H: N T**

Methodology of reconstruction

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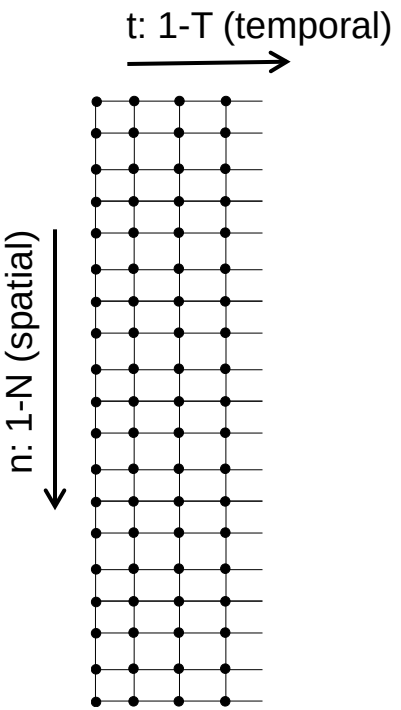
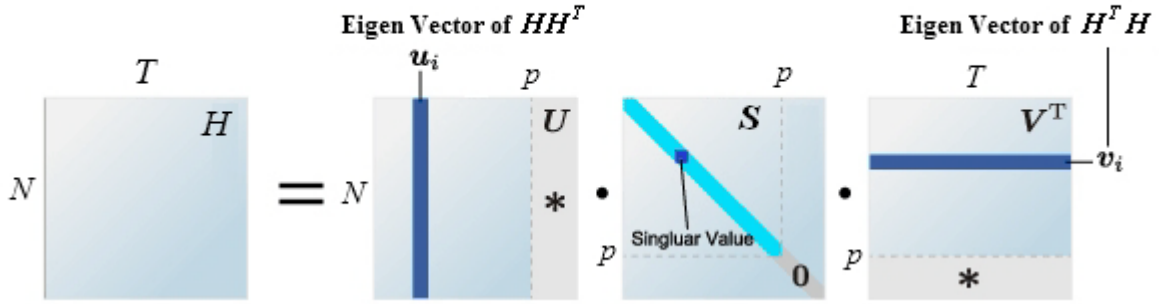


$$\underset{N\times T}{H} = \underset{N\times N}{U}\ \underset{N\times T}{S}\ \underset{T\times T}{V^T}$$



(Modified from internet resources)

Methodology of reconstruction

- The basic ideas
  - Filter out noise
  - Capture spatial pattern
  - Fill data gaps

$$\underset{N\times p}{U'}$$

- Review of previous methods
  - "Reduced SVD"

$$\underset{N\times T}{H} \cong \underset{N\times p}{U'}\ \underset{p\times p}{S}\ \underset{p\times T}{V^T} \longrightarrow \underset{N\times T}{H} \cong \underset{N\times p}{U'}\ \underset{p\times T}{A} \longrightarrow \underset{N\times 1}{h(t)} \cong \underset{N\times p}{U'}\ \underset{p\times 1}{\alpha(t)}$$

  - Reconstruction: going beyond T
    - At time t', only R observations, data at N-R points need to be reconstructed

$$\underset{R\times 1}{hr(t')} \cong \underset{R\times p}{Ur'}\ \underset{p\times 1}{\alpha(t')} \longrightarrow \underset{p\times 1}{\alpha(t')} \longrightarrow \underset{N\times 1}{h(t')} \cong \underset{N\times p}{U'}\ \underset{p\times 1}{\alpha(t')}$$

    - Alternative names in climate studies: empirical orthogonal functions (EOFs), reduced space optimal interpolation **(Smith et al., 1996; Kaplan et al., 2000; Church et al., 2004)**

Methodology of reconstruction

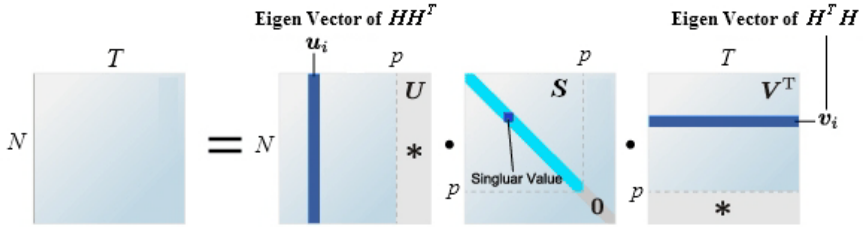
❑ Our method of data reconstruction

- Why not “reduced SVD” (Church et al., 2004) ? → different tasks
  - Data gap vs data famine
  - Construction of spatial pattern
  - Uncertainty issue

$$h(t)_{N\times 1} \cong U'_{N\times p}\, \alpha(t)_{p\times 1}$$

- Ideas in “reduced SVD” to serve in the development of new methods
  - Certain spatial relationships do not change over time

$$U'_{N\times p}$$



  - Magnitudes of major spatial components can be calibrated during reconstruction

❑ The basic ideas

- Filter out noise
- Capture spatial pattern
- Fill data gaps

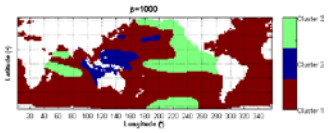
Methodology of reconstruction

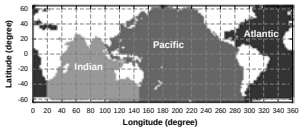
❑ Our approach to realize the basic ideas

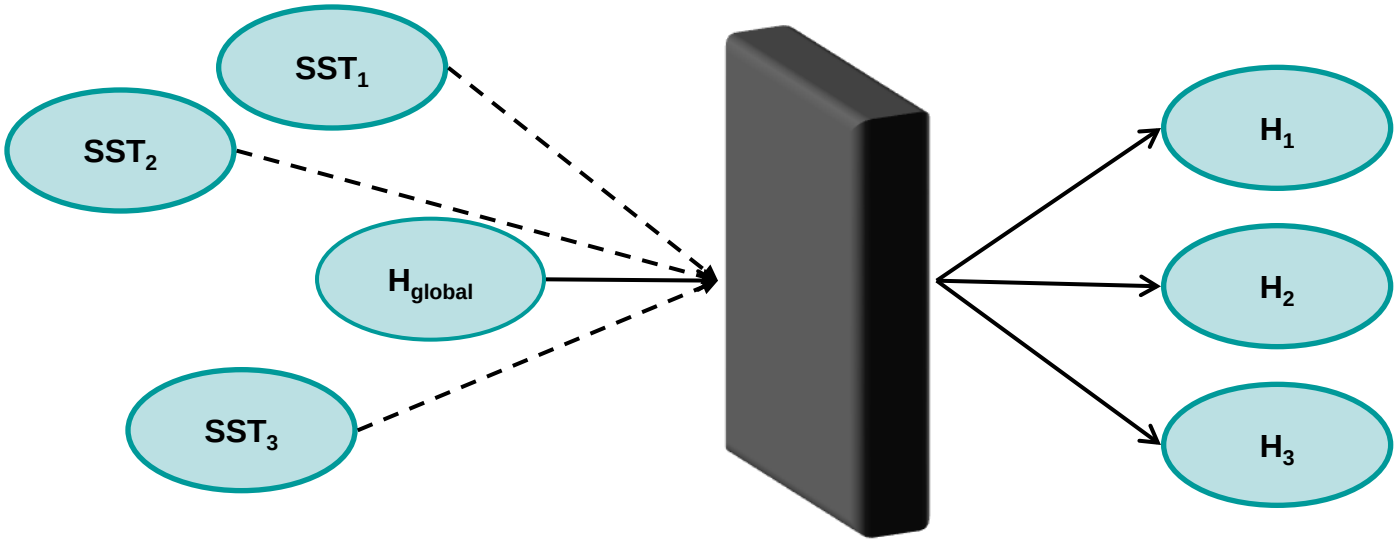
- Filter out noise → clustering and subsequent spatial averaging within clusters
- Capture spatial pattern → artificial neural network (NN)
- Fill data gaps → utilizing global mean sea level and spatial SST data

❑ Neural network architecture

- Starting from the “black box” perspective







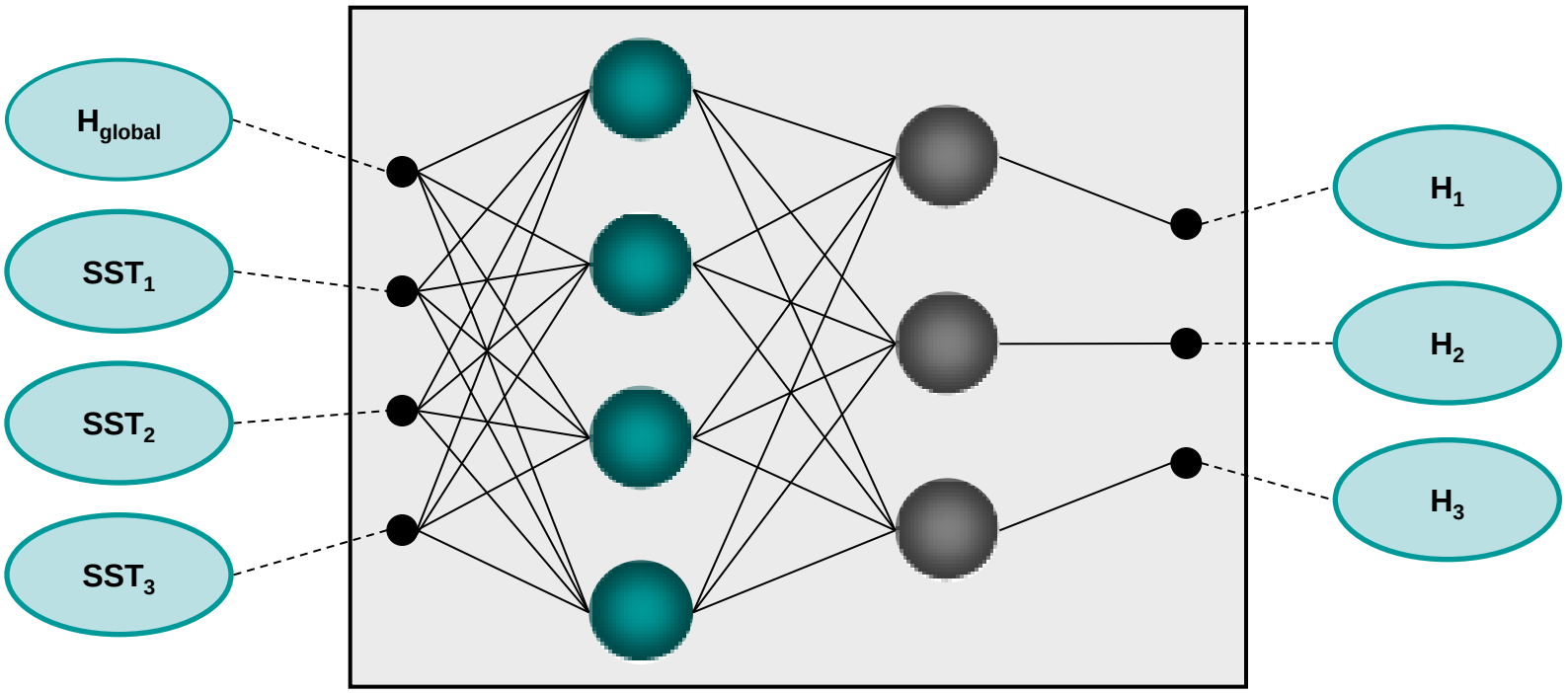
Temporal coverage: 1880-2001

Temporal coverage: 1952-2001

Methodology of reconstruction

❑ Neural network architecture

- Inside the “black box”
  - Type of neural network: feedforward
  - Neurons: layer and number
  - Within neuron: weight, bias, transfer function
  - Pre- and post- processing



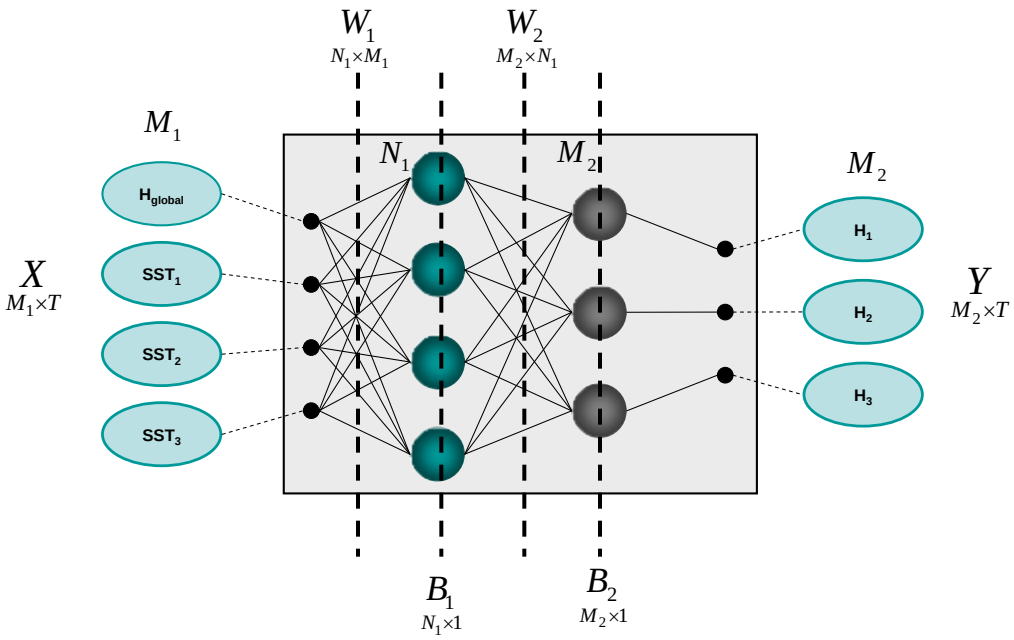
Methodology of reconstruction

❑ Mathematics of NN

- Weights and biases

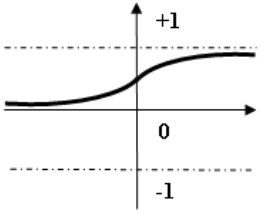
$$A_{1} = TransFcn1\left(W_1 X + B_1\right)$$
$$N_1 \times T$$

$$Y = TransFcn2\left(W_2 A_1 + B_2\right)$$
$$M_2 \times T$$

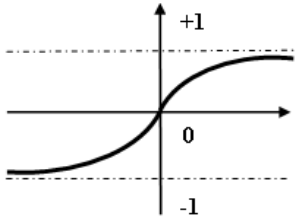


- Transfer functions

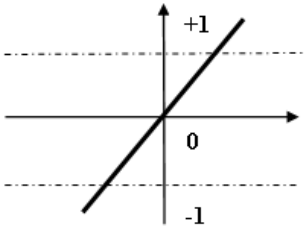
$$\text{Log-Sigmoid}: y = \frac{1}{1+e^{-x}}$$



$$\text{Tan-Sigmoid}: y = \frac{2}{1+e^{-2x}} - 1$$



$$\text{Linear transfer}: y = x$$



Methodology of reconstruction

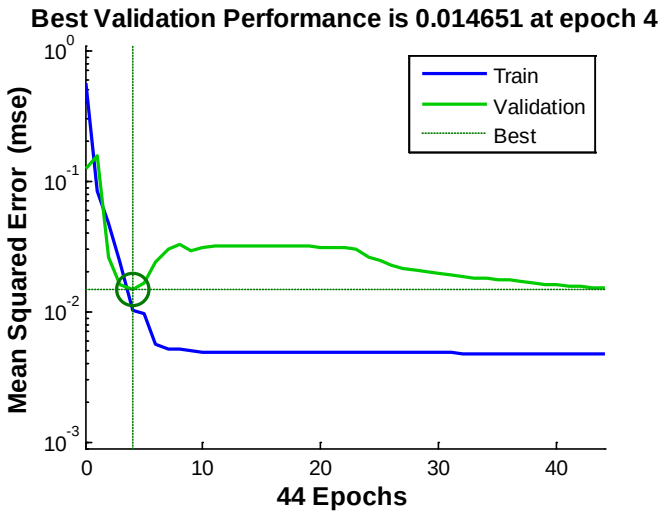
❑ Training and validating NN

- NN training is first an optimization problem
  - Gradient descent and related
  - Conjugate gradient and related
  - Levenberg-Marquardt algorithm
  - Other

$$\beta_{k+1} = \beta_k - \left[\mathbf{J}_k{}^{\mathrm{T}} \mathbf{J}_k + \lambda \mathbf{I} \right]^{-1} \mathbf{J}_k{}^{\mathrm{T}} \left[Y - f\left(\beta_k\right) \right]$$

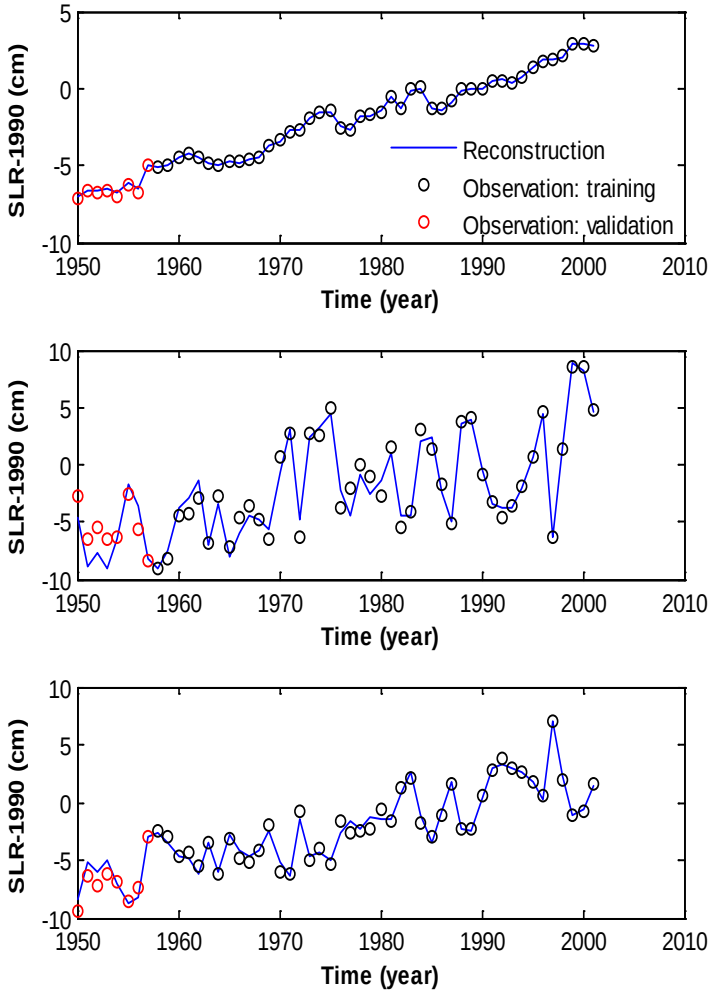
$$\beta_{k+1} = \beta_k - \left[\mathbf{J}_k{}^{\mathrm{T}} \mathbf{J}_k + \lambda \mathrm{diag}\left(\mathbf{J}_k{}^{\mathrm{T}} \mathbf{J}_k \right) \right]^{-1} \mathbf{J}_k{}^{\mathrm{T}} \left[Y - f\left(\beta_k\right) \right]$$

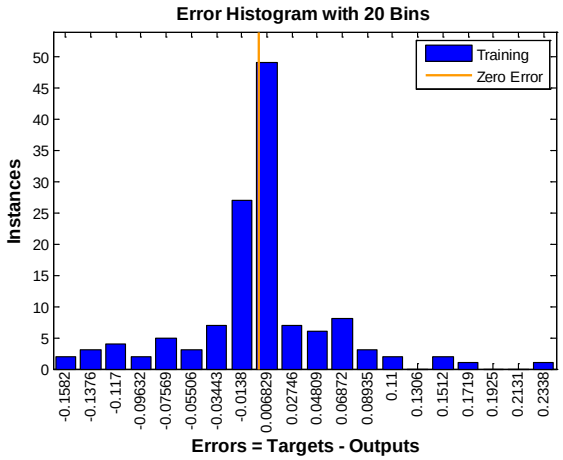
- Validating NN to improve generalization
  - The best training vs. the best generalization

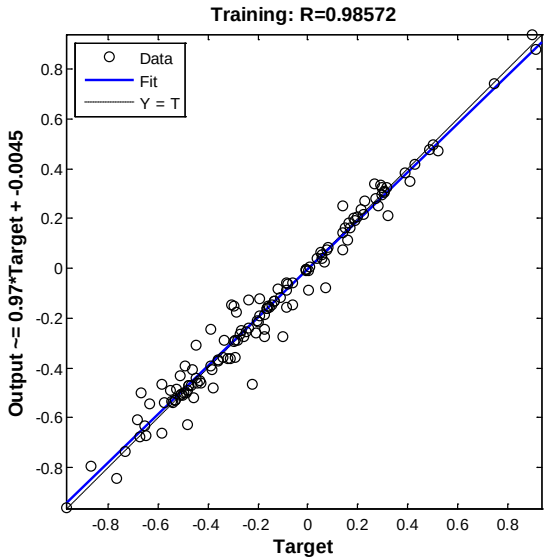


Results of reconstruction

- Training and validation: example



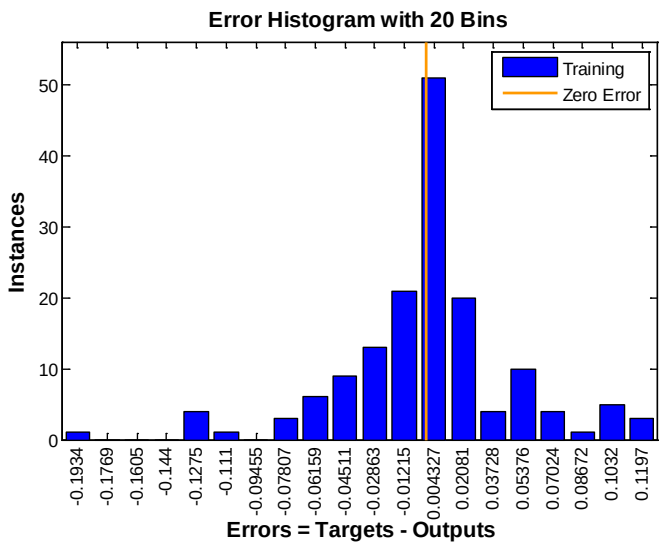




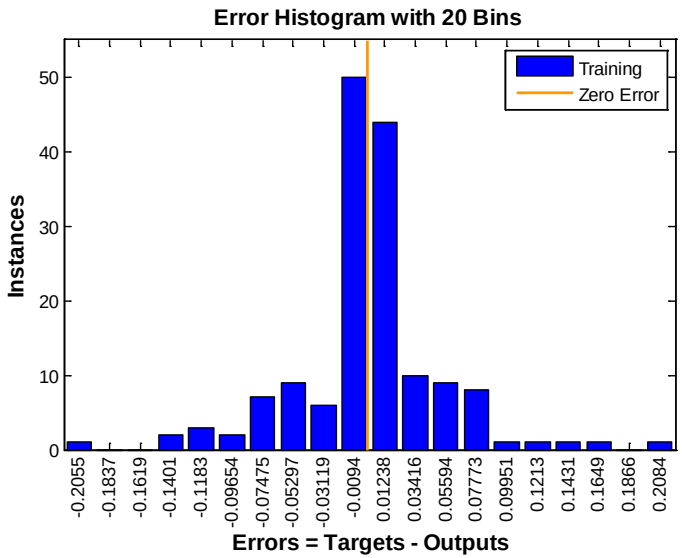
Results of reconstruction

- Issue 1: local minimum + initial weights/biases

"zero" initialization



random [-1, 1] initialization

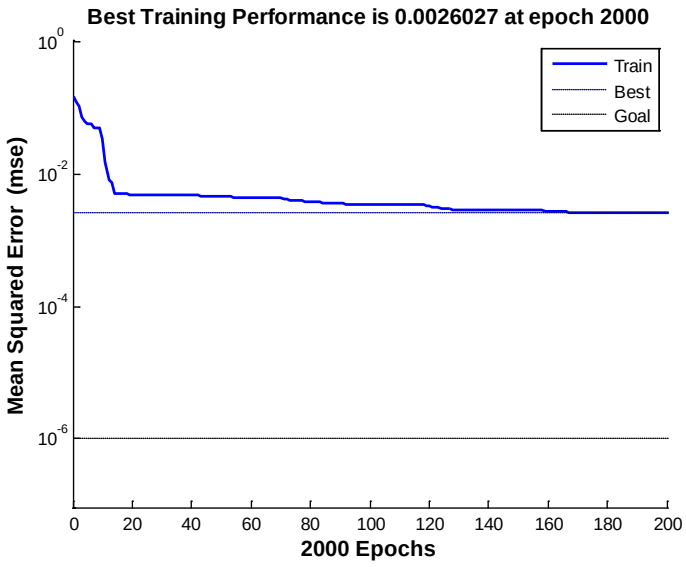


- Solution:
  - Multiple trainings with random initial weights/biases (1000 reps)

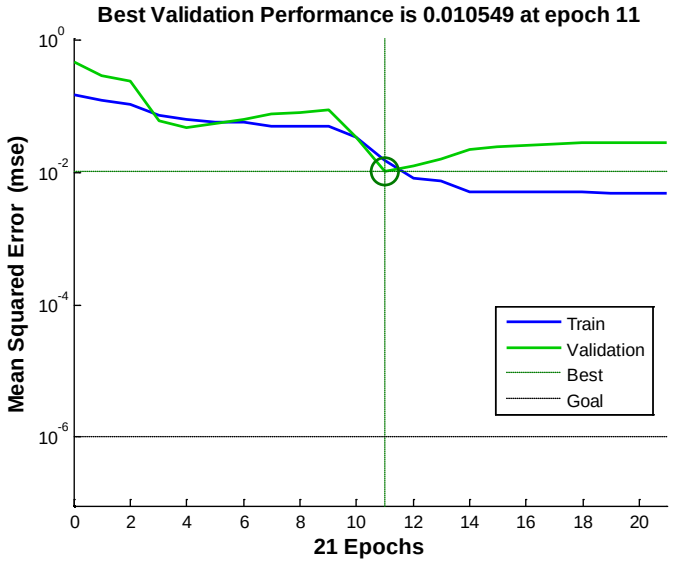
Results of reconstruction

- Issue 2: generalization

Training without validation check



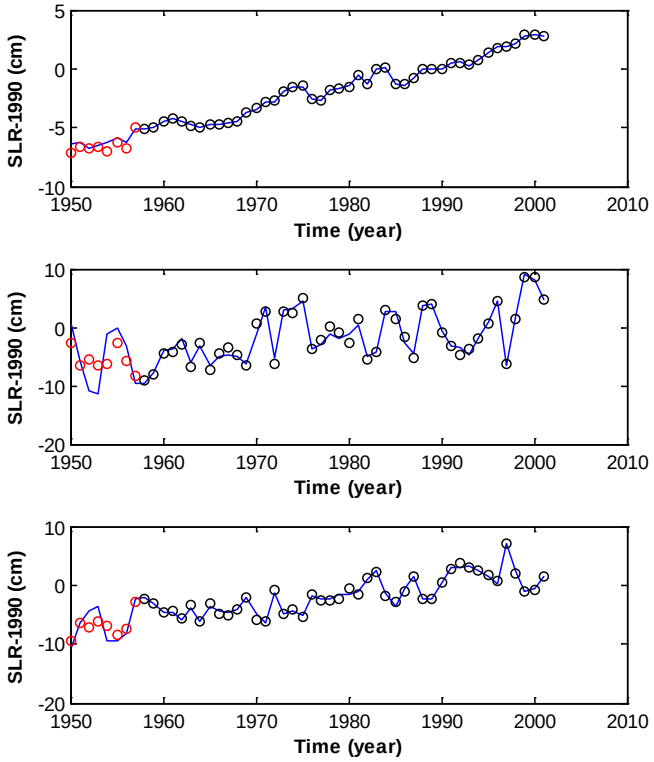
Training with validation check



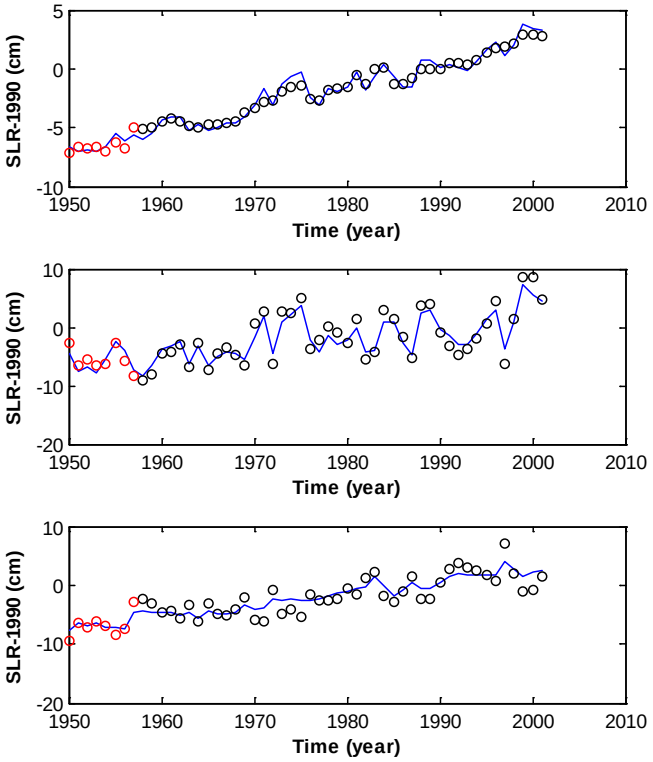
Results of reconstruction

- Issue 2: generalization

Training without validation check



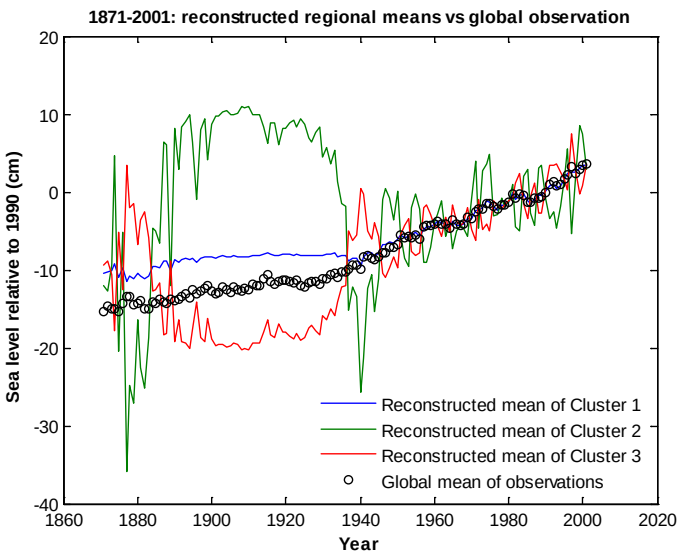
Training with validation check



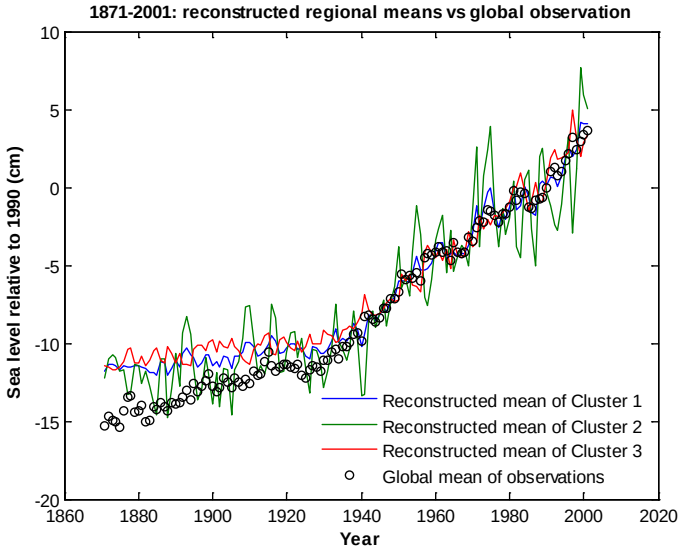
Results of reconstruction

- Issue 2: generalization

Training without validation check



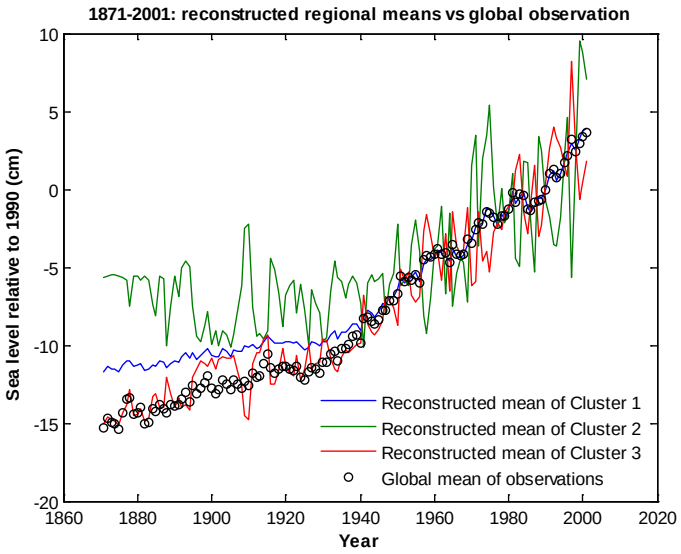
Training with validation check



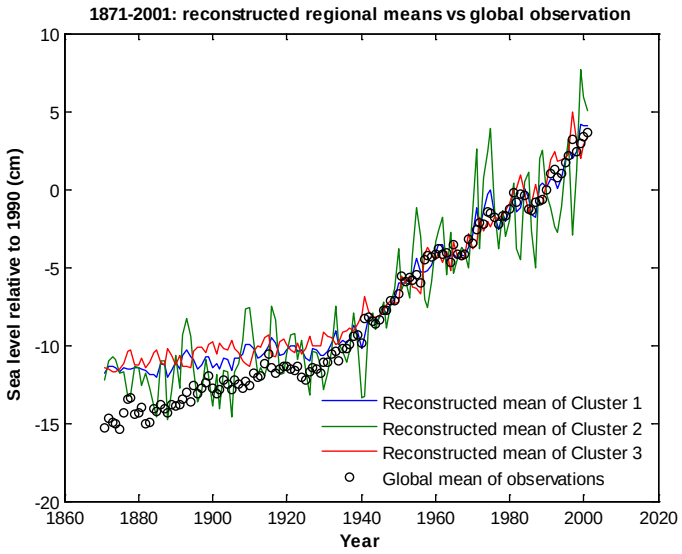
Results of reconstruction

- Issue 2: generalization

Validation dataset used as training data



Training with validation check

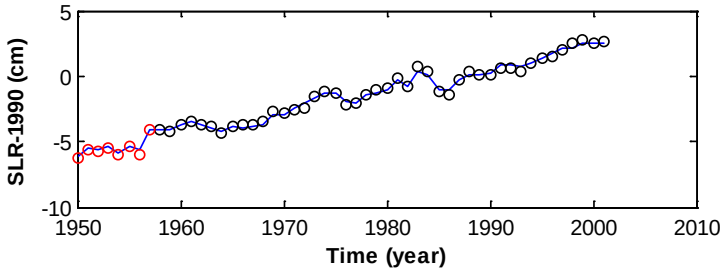


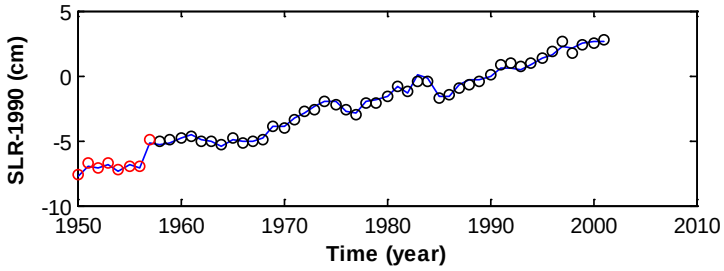
- Solution:
  - Training with validation check (15%)

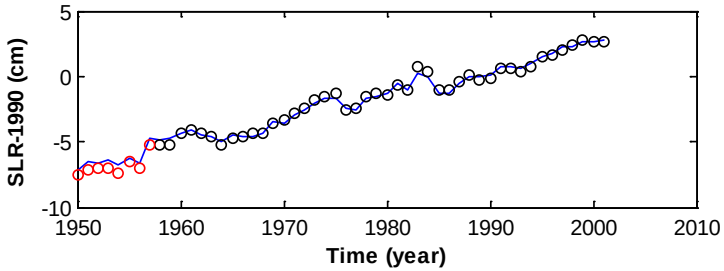
Results of reconstruction

- Impact of region division

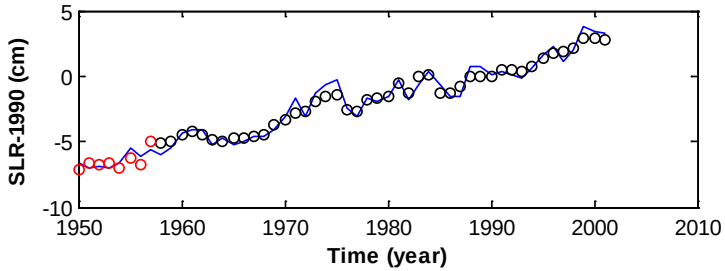
Division based on ocean basins

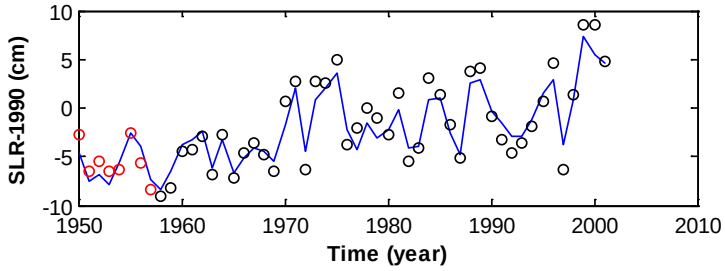


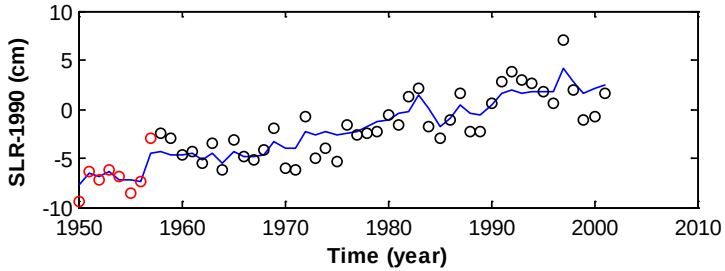




Division based on clustering



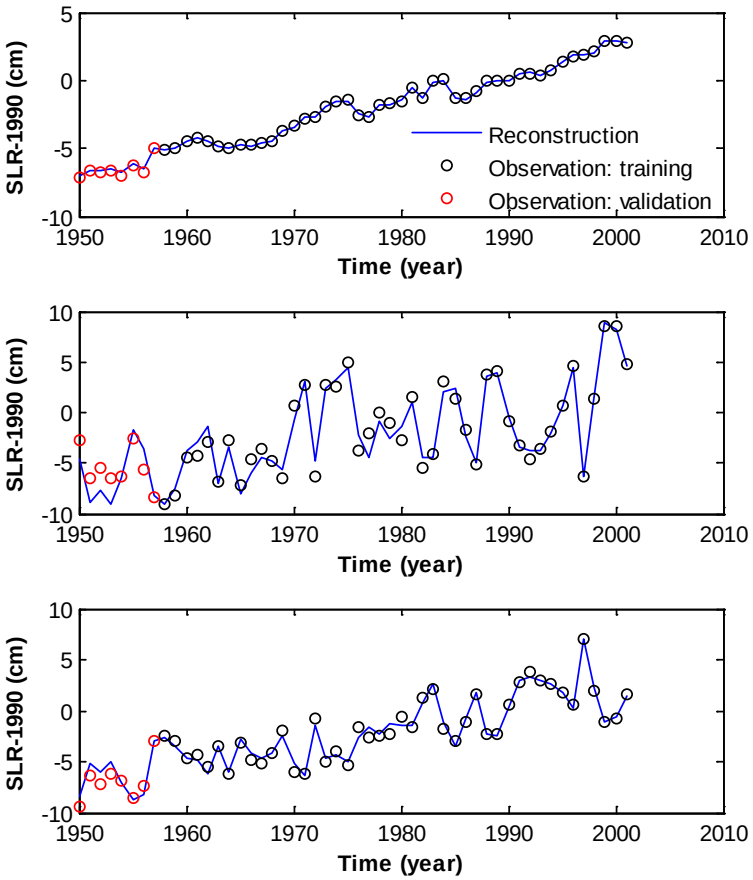




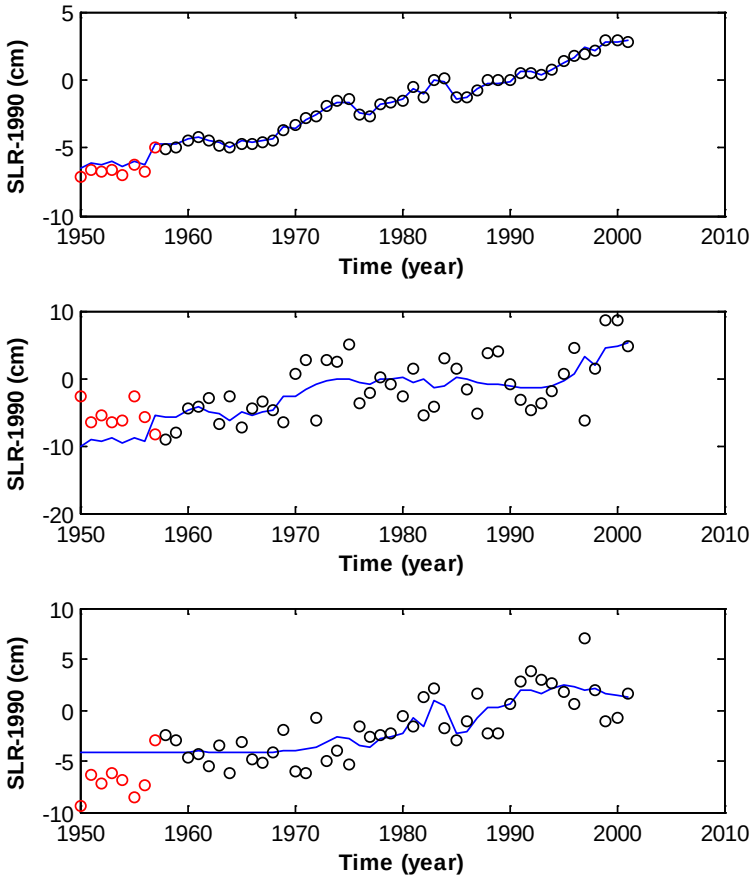
Results of reconstruction

- Impact of SST as input: clustering

H_{global} and 3 SST's as inputs



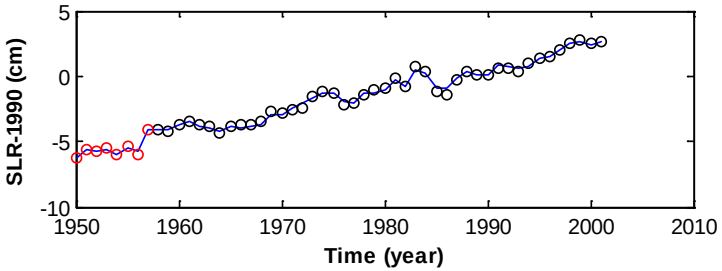
H_{global} as the only input

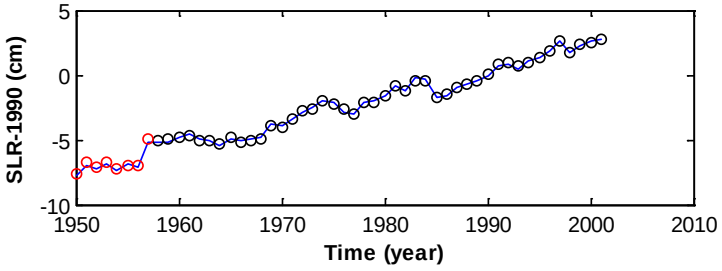


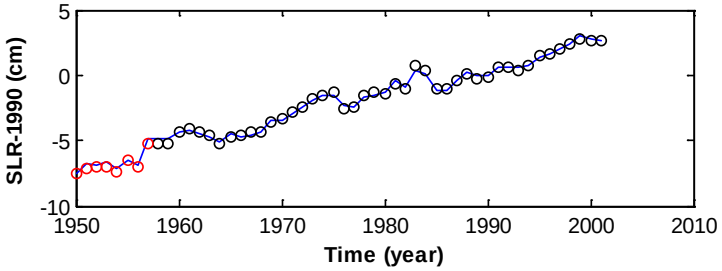
Results of reconstruction

- Impact of SST as input: ocean basin

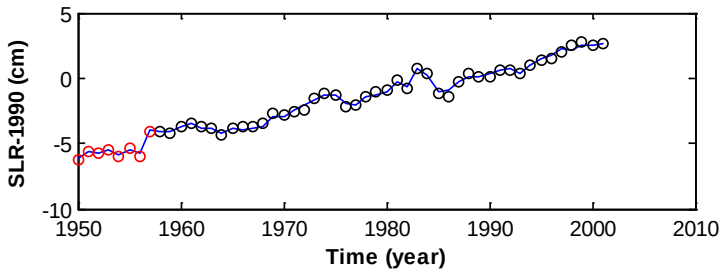
H_{global} and 3 SST's as inputs

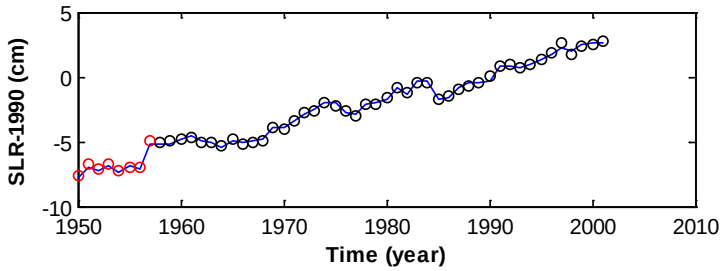


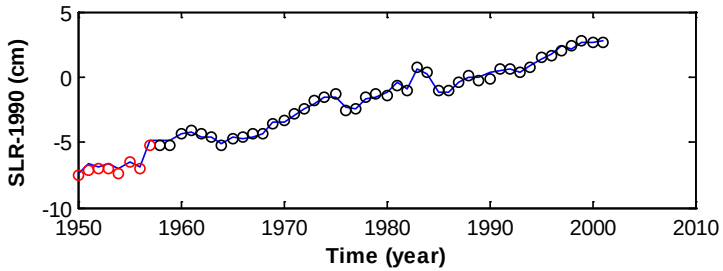




H_{global} as the only input

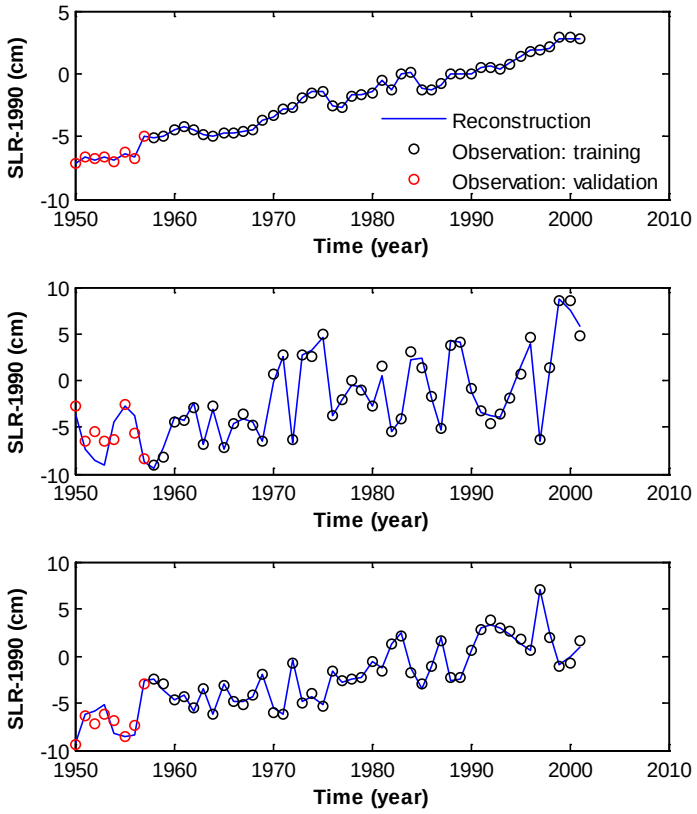


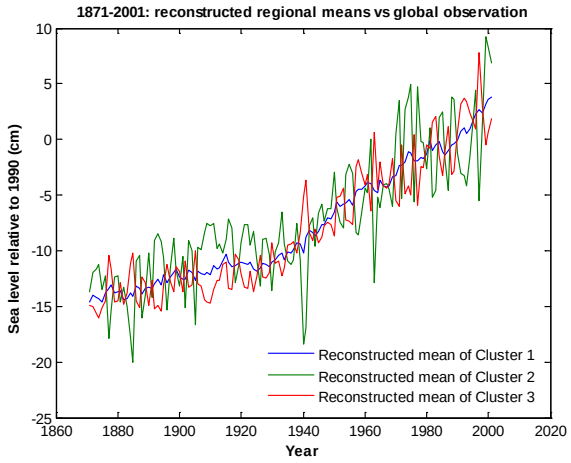


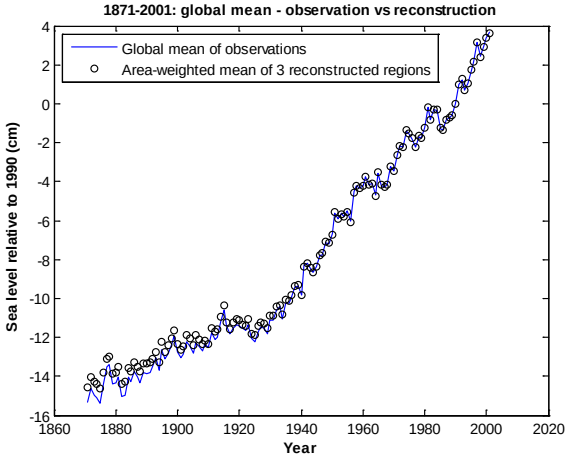


Results of reconstruction

- Final results: clustering

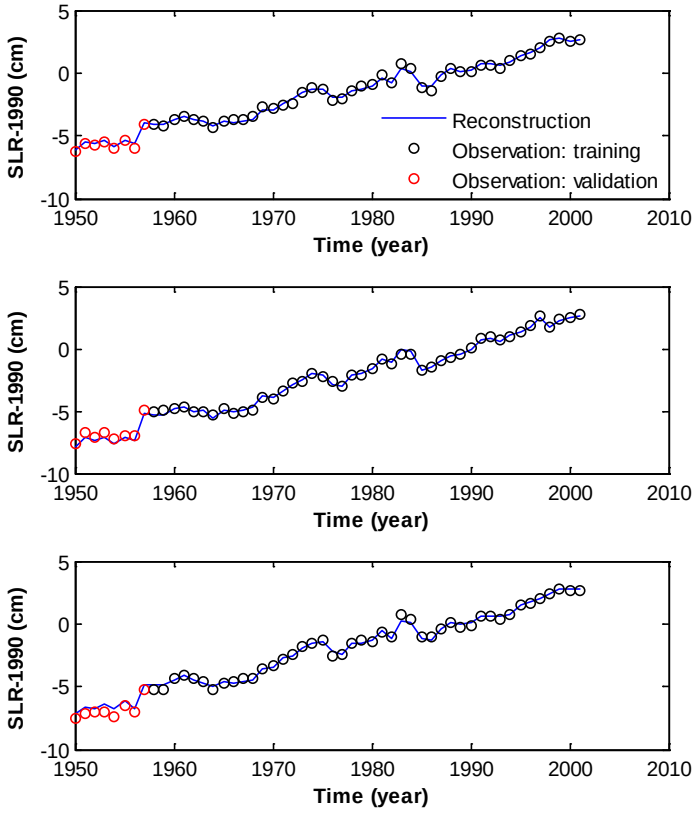


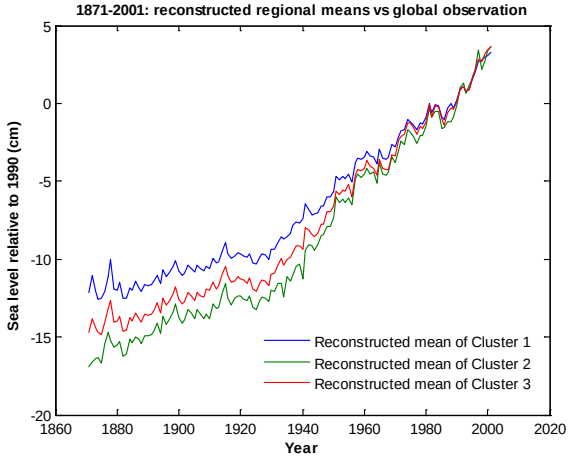


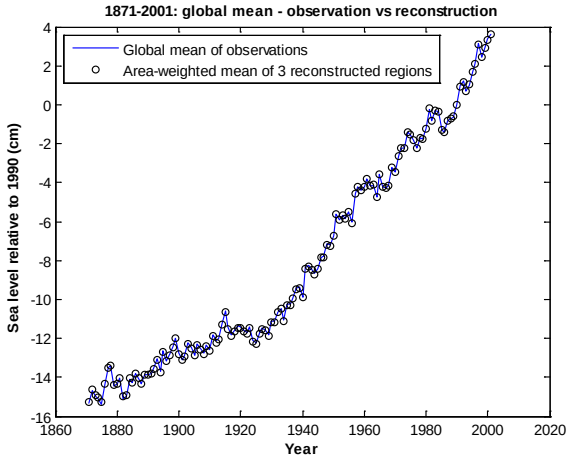


Results of reconstruction

- Final results: ocean basin







The End

Thanks!